RESEARCH ON THE LAWS OF KEPLER'S PLANETARY MOVEMENT

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CLASS AND ADDRESS: 2Y - MATHEMATICAL SCIENTIFIC

OUR TARGET: *This article is intended for teenagers who have at least attended the third year of high school studies or have already dealt with the figure of the ellipse. In particular, scientific and classical high school.*

OUR GOALS: *The goal of the article is to show all interested students everything that lies behind a seventeenth century genius, John Kepler. We believe that in order to make the work more pleasant both for us and for the reader, it is best to use understandable and simple language at the same time, so as to be able to focus on the main concepts. Furthermore, it is also important to visualize concepts not only through formulas, but also with figures.*

Planetary Physics: Kepler's Laws of Planetary Motion

Kepler's three laws describe how planetary bodies orbit the Sun.

- Firstly, they describe how planets move in elliptical orbits with the Sun as a focus also expressible as $A = \pi a b$;
- Secondly, how a planet covers the same area of space in the same amount of time no matter where it is in its orbit, also expressible as $\frac{dA}{dt} = \frac{abn}{2} = \frac{\pi a b}{T}$;
- Thirdly, how a planet's orbital period is proportional to the size of its orbit (its semi-major axis), also expressible as $\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2} \approx 7{,}496 \times 10^{-6} \frac{AU^3}{days^2}$.

Let's explore the process that Johannes Kepler undertook when he formulated his three laws of planetary motion.

Kepler's Life and Discoveries

The planets orbit the Sun in a counterclockwise direction as viewed from above the Sun's north pole, and the planets' orbits all are aligned to what astronomers call the ecliptic plane.

The story of our greater understanding of planetary motion could not be told if it were not for the work of a German mathematician named Johannes Kepler. Kepler lived in Graz, Austria during the tumultuous early 17th century. Due to religious and political difficulties common during that era, Kepler was banished from Graz on August 2nd, 1600. Fortunately, an opportunity to work as an assistant for the famous astronomer Tycho Brahe presented itself and the young Kepler moved his family from Graz 300 miles across the Danube River to Brahe's home in Prague. Tycho Brahe is credited with the most accurate astronomical observations of his time and was impressed with the studies of Kepler during an earlier meeting. However, Brahe mistrusted Kepler, fearing that his bright young intern might eclipse him as the premier astronomer of his day. He, therefore, led Kepler to see only part of his voluminous planetary data.

He set Kepler, the task of understanding the orbit of the planet Mars, the movement of which fit problematically into the universe as described by Aristotle and Ptolemy. It is believed that part of the motivation for giving the Mars problem to Kepler was Brahe's hope that its difficulty would occupy Kepler while Brahe worked to perfect his own theory of the solar system, which was based on a geocentric model, where the earth is the center of the solar system. Based on this model, the planets Mercury, Venus, Mars, Jupiter, and Saturn all orbit the Sun, which in turn orbits the earth. As it turned out, Kepler, unlike Brahe, believed firmly in the Copernican model of the solar system known as heliocentric, which correctly placed the Sun at its center. But the reason Mars' orbit was problematic was because the Copernican system incorrectly assumed the orbits of the planets to be circular.

After much struggling, Kepler was forced to an eventual realization that the orbits of the planets are not circles, but were instead the elongated or flattened circles that geometers call ellipses, and the particular difficulties Brahe hand with the movement of Mars were due to the fact that its orbit was the most elliptical of the planets for which Brahe had extensive data. Thus, in a twist of irony, Brahe unwittingly gave Kepler the very part of his data that would enable Kepler to formulate the correct theory of the solar system, banishing Brahe's own theory. Since the orbits of the planets are ellipses, let us review three basic properties of ellipses. The first property of an ellipse: an ellipse is defined by two points, each called a focus, and together called foci. The sum of the distances to the foci from any point on the ellipse is always a constant.

The second property of an ellipse: the amount of flattening of the ellipse is called the eccentricity. The flatter the ellipse, the more eccentric it is. Each ellipse has an eccentricity with a value between zero, a circle, and one, essentially a flat line, technically called a parabola.

The third property of an ellipse: the longest axis of the ellipse is called the major axis, while the shortest axis is called the minor axis. Half of the major axis is termed a semi-major axis. Knowing then that the orbits of the planets are elliptical, Johannes Kepler formulated three laws of planetary motion, which accurately described the motion of comets as well.

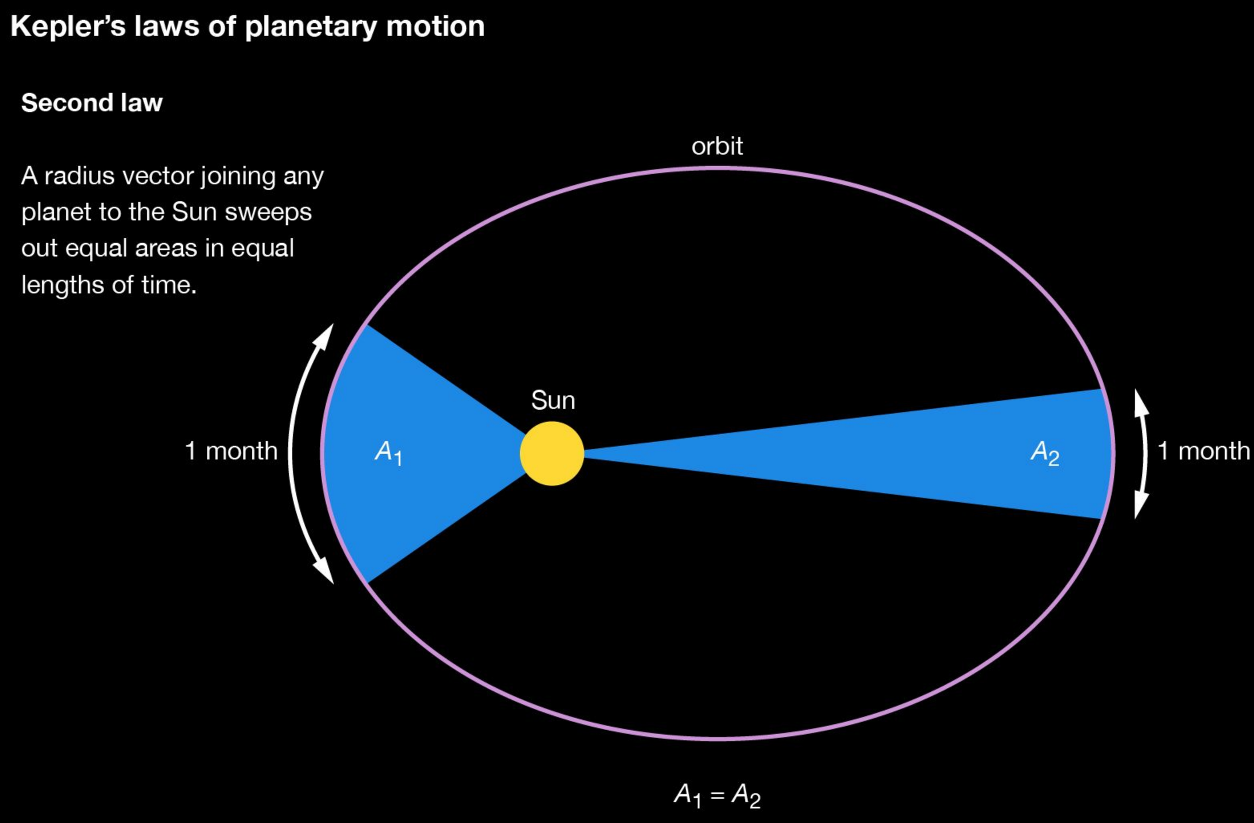
Let's see Kepler's **First Law**: each planet's orbit about the Sun is an ellipse. The Sun's center is always located at one focus of the orbital ellipse. The Sun is at one focus. The planet follows the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit

Kepler's **Second Law**: the imaginary line joining a planet and the Sun sweeps equal areas of space during equal time intervals as the planet orbits. Basically, that planets do not move with constant speed along their orbits. Rather, their speed varies so that the line joining the centers of the Sun and the planet sweeps out equal parts of an area in equal times. The point of nearest approach of the planet to the Sun is termed perihelion. The point of greatest separation is aphelion, hence by Kepler's Second Law, a planet is moving fastest when it is at perihelion and slowest at aphelion.

Kepler's **Third Law**: the squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axes of their orbits. Kepler's Third Law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus we find that Mercury, the innermost planet, takes only 88 days to orbit the Sun. The earth takes 365 days, while Saturn requires 10,759 days to do the same. Though Kepler hadn't known about gravitation when he came up with his three laws, they were instrumental in Isaac Newton deriving his theory of universal gravitation, which explains the unknown force behind Kepler's Third Law.

Kepler and his theories were crucial in the better understanding of our solar system dynamics and as a springboard to newer theories that more accurately approximate our planetary orbits. The first two laws of planetary motion were announced by the astronomer in the year 1609, while the last one nearly a decade later, in 1618. Kepler himself never numbered these laws or specially distinguished them from his other discoveries.

Let's analyze the demonstration of Kepler's second law: A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.



And now, the demonstration of Kepler's third law: The squares of the sidereal periods (P) of the planets are directly proportional to the cubes of their mean distances (d) from the Sun.

Kepler's laws of planetary motion

Third law

The squares of the sidereal periods (*P*) of the planets are directly proportional to the cubes of their mean distances (*d*) from the Sun.

$$P \times P = k\,(d \times d \times d)$$

$$P^2 = kd^3$$

$$\frac{P^2}{d^3} = k$$

where *k* is a constant

planet	period (*P*, year)	period squared	mean distance (*d*, AU)	mean distance cubed	P^2/d^3
Mercury	0.24	0.06	0.39	0.06	0.99
Venus	0.62	0.38	0.72	0.38	1.02
Earth	1.00	1.00	1.00	1.00	1.00
Mars	1.88	3.53	1.52	3.51	1.01
Jupiter	11.86	140.66	5.20	140.61	1.00
Saturn	29.46	867.89	9.58	879.22	0.99
Uranus	84.01	7057.68	19.20	7077.89	1.00
Neptune	164.80	27159.04	30.10	27270.90	1.00

… In few words (and some interesting facts)

Kepler's three laws of planetary motion can be stated as follows: All planets move about the Sun in elliptical orbits, having the Sun as one of the foci. A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time. The squares of the sidereal periods (of revolution) of the planets are directly proportional to the cubes of their mean distances from the Sun. Knowledge of these laws, especially the second (the law of areas), proved crucial to Sir Isaac Newton in 1684–85, when he formulated his famous law of gravitation between Earth and the Moon and between the Sun and the planets, postulated by him to have validity for all objects anywhere in the universe. Newton showed that the motion of bodies subject to central gravitational force need not always follow the elliptical orbits specified by the first law of Kepler but can take paths defined by other, open conic curves; the motion can be in parabolic or hyperbolic orbits, depending on the total energy of the body. Thus, an object of sufficient energy—e.g., a comet—can enter the solar system and leave again without returning. From Kepler's second law, it may be observed further that the angular momentum of any planet about an axis through the Sun and perpendicular to the orbital plane is also unchanging.

All About Astronomy

Brian Greene demonstrates how Newton's law of gravitation determines the trajectories of the planets and explains the patterns in their motion found by Kepler.
The usefulness of Kepler's laws extends to the motions of natural and artificial satellites, as well

as to stellar systems and extrasolar planets. As formulated by Kepler, the laws do not, of course, take into account the gravitational interactions (as perturbing effects) of the various planets on each other. The general problem of accurately predicting the motions of more than two bodies under their mutual attractions is quite complicated; analytical solutions of the three-body problem are unobtainable except for some special cases. It may be noted that Kepler's laws apply not only to gravitational but also to all other inverse-square-law forces and, if due allowance is made for relativistic and quantum effects, to the electromagnetic forces within the atom.

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